國立清華大學數學系學術演講
NTHU MATH Colloquium

講題　A Self-Adaptive Theta Scheme using Discontinuity Aware Quadrature for Solving Conservation Laws

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地點　Room 101, General Building III

茶會　15:30, R707

Abstract

We present a *discontinuity aware quadrature* (DAQ) rule, and use it to develop implicit *self-adaptive theta* (SATh) schemes for the approximation of scalar hyperbolic conservation laws. Our SATh schemes require the solution of a system of two equations, one controlling the cell averages of the solution at the time levels, and the other controlling the space-time averages of the solution. These quantities are used within the DAQ rule to approximate the time integral of the hyperbolic flux function accurately, even when the solution may be discontinuous somewhere over the time interval. The result is a finite volume scheme using the theta time stepping method, with theta defined implicitly (or self-adaptively). Two schemes are developed, SATh-up for a monotone flux function using simple upstream stabilization, and SATh-LF using the Lax-Friedrichs numerical flux. We prove that DAQ is accurate to second order when there is a discontinuity in the solution and third order when it is smooth. We prove that SATh-up is unconditionally stable (provided that theta is set to be at least 1/2), satisfies the maximum principle and is total variation diminishing under appropriate monotonicity and boundary conditions. General flux functions require the SATh-LF scheme, so we assess its accuracy through numerical examples in one and two space dimensions. These results suggest that SATh-LF is also stable and satisfies the maximum principle (at least at reasonable CFL numbers). Compared to solutions of finite volume schemes using Crank-Nicolson and backward Euler time stepping, SATh-LF solutions often approach the accuracy of the former but without oscillation, and they are numerically less diffuse than the later.

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